

AC Machines: Synchronous Machines

Lecture Notes

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A.C. MACHINES

Text Book : Electric Energy Conversion and Transmission

By : Syed . A. Nasar .

Reference Books:

1. Electrical Machines and their applications

By : John Hindmarsh.

2. Alternating Current Machines

By : MG Say

3. Electric Machinery

By : A.E. Fitzgerald & Charles Kingsley.

4. $\text{الهندسة الكهربائية} - \text{محركات التيار المتردد} - \text{د. طارق محمد العبدالله}$

I. Synchronous Machines : Steady State operation.

A synchronous machine is an ac machine whose speed under steady-state conditions is proportional to the frequency of the current in its armature. The magnetic field created by the armature currents rotates at the same speed as that created by field current on the rotor (which is rotating at synchronous speed), and a steady torque results.

a. Elementary Synchronous Machines :

A very much simplified a.c. synchronous generator is shown in Fig.1. with Salient poles.

* The armature winding of a synchronous machine is on the stator

* The field winding is on the rotor

* The field winding is excited by direct current conducted to it by means of carbon brushes bearing on slip rings.

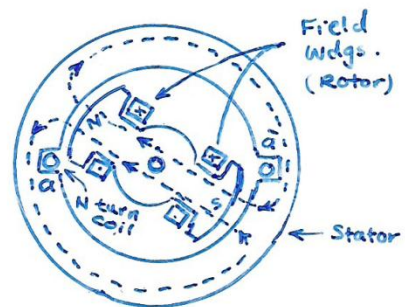


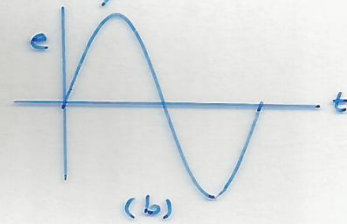
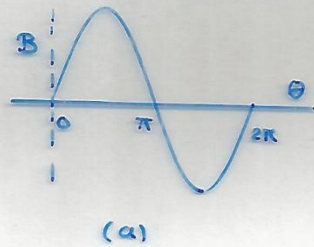
Fig.1

* The advantages of such construction is:

1. Heavy current may drawn from the stator windings
2. low-power field winding on the rotor minimize electrical and mechanical problems.

The rotor is turned at a constant speed by a source of mechanical power (prime-mover) connected to its shaft. Flux paths are shown by dotted lines.

The radial distribution of the air-gap flux density B is shown in Fig. 2(a) as a function of the space angle θ around the air-gap periphery.



As the rotor revolves, the flux waveform sweeps the coil sides a and $-a$. The resulting coil voltage (Fig. 2(b)) is a time function having the same waveform as the spatial distribution B . The induced voltage e has a complete cycle for each revolution of the two pole machine. Its frequency in Hz is the same as the speed of the rotor in revolution per second, i.e., the electrical frequency is synchronized with the mechanical speed, and this is the reason why we call it synchronous machine.

Thus a 2-pole synch. m/c must revolve at 3000 rpm to produce a 50 Hz voltage.

- Multi-pole machines: In general, many synch. m/cs have more than 2-pole. For example Fig. 3 shows a 4-pole single-phase alternator (synch. m/c). The field coils are connected so that the poles are alternate north and south polarity. The armature winding consists of two coils $a_1, -a_1$ and $a_2, -a_2$ connected in series by their end connections. The frequency of the induced e.m.f is now.

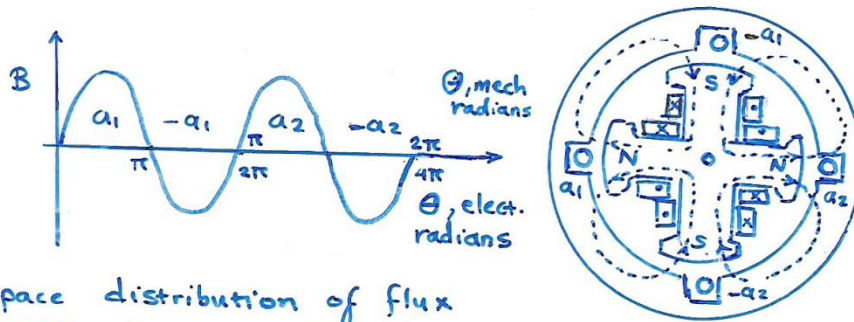


Fig. 3. space distribution of flux density in a 4-pole synch. generator

4-pole synch. generator

In order to understand the action of p-pole m/c, it is easier to consider a single pair of poles and to recognize that the electric, magnetic, and mechanical conditions associated with every other pair are repetitions of those for the pair under consideration. Hence, one pair of poles in a p-pole machine or one cycle of flux distribution equals 360 electrical degrees or 2π electrical radians. Since there are $P/2$ complete wavelengths or cycles in one complete revolution, hence

$$\theta_e = \frac{P}{2} \theta_m$$

θ_e = electrical angle.
 θ_m = mechanical angle.

The coil voltage of a p-pole m/c passes through a complete cycle every time a pair of poles sweeps by, or $P/2$ times each revolution. The frequency of the voltage wave is therefore:

$$f = \frac{P}{2} \cdot \frac{n}{60} \text{ Hz}$$

where n is the mechanical speed in revolution per minute. and $n/60$ is the speed in rev. per second. The radian frequency ω of the voltage wave is

$$\omega = \frac{P}{2} \omega_m$$

where ω_m is the mechanical speed in radians per second.

b. Elementary synch. machine with Cylindrical-rotor :

Fig. 4 shows a cylindrical rotor 2-pole synch. m/c. The field winding is a distributed winding placed in slots and arranged to produce an approximately sinusoidal 2-pole field.

- * Usually, cylindrical rotor are used for high speed machines like turbogenerator.
- * For low speed machines like Hydroelectric and diesel-electric generators, salient poles rotors are used.

* Most of power systems in Iraq operate at a frequency of 50 Hz.

c. Stator construction

With very few exceptions, synch. generators are 3-phase machines because of the advantages of 3-phase systems for generation, transmission and heavy-power utilization. For the production of a set of three voltages phase-displaced by 120° in time, three coils phase-displaced 120° electrical degrees in space must be used. An elementary 3-ph 2-pole m/c with one coil per phase is shown in Fig. 5.

These coils are form the armature of the m/c and usually carried by the stator.

Often, more than one coil or winding is on the stator.

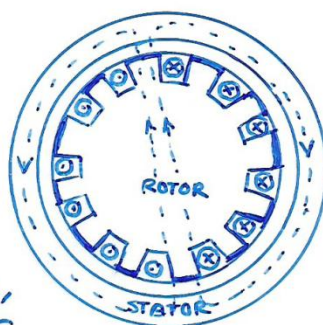


Fig. 4

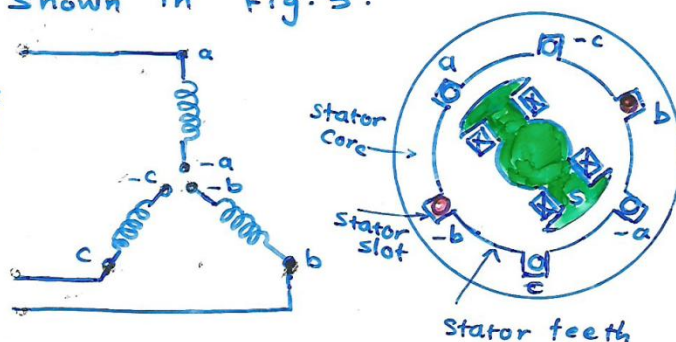


Fig. 5

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Typical three-phase windings: (i) single-layer winding:

A typical 3-ph winding of 24 conductors is shown in Fig. 6, A, -a are the ends of the first phase b, -b those of the second, and C, -C those of the third.

When the centre line of coil A coincides with that of the north pole, the induced voltage will be zero.

Now supposing the poles fixed, and the winding moving from right to left, the induced ~~induced~~ voltage in the first phase will rise in value from zero in an anti-clockwise direction, that is, from -a

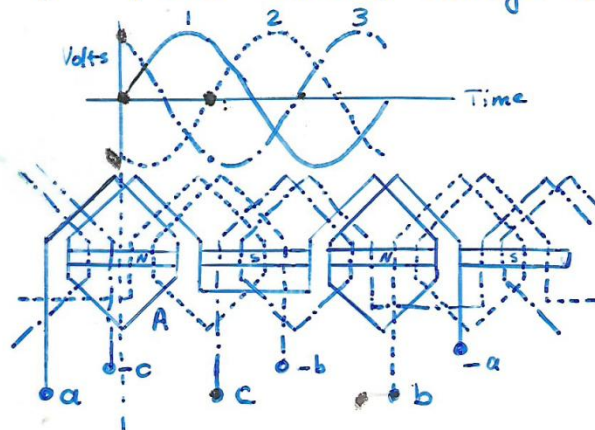


Fig. 6

to a through the first-phase section of the winding. This direction will be maintained through a movement equal to the pole pitch, the voltage rising from zero to a maximum, then back to zero.

The second-phase section has to move 120° electrical degrees before attaining the same voltage and direction that the first phase section had at the instant represented in the figure, while the third-phase section has to move through 240 electrical degrees to attain the same result.

Therefore, reckoning from -a to a, -b to b, -c to c, the respective in phases are:

$$e_1 = E_m \sin \omega t$$

$$e_2 = E_m \sin (\omega t - 120)$$

$$e_3 = E_m \sin (\omega t - 240)$$

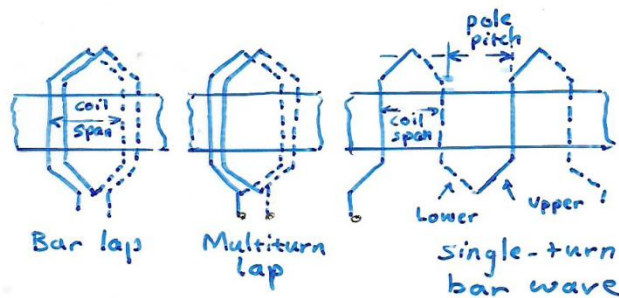
Double-layer windings:

The armatures of nearly all synchronous generators and motors, and of most induction motors above a few kilowatts, are wound with double-layer windings. (see Fig. 6-4. PP-179 in the textbook).

* Windings with a whole number of slots per pole ($g = \frac{S}{2P}$ an integer) are called integral slot wds.

* When g is fractional the windings are termed fractional-slot windings. Also called fractional pitch or (skewed) winding.

Type of double-layer windings: There are two main type of double-layer windings; namely lap and wave windings. Lap may be either Bar lap or Multiturn lap.

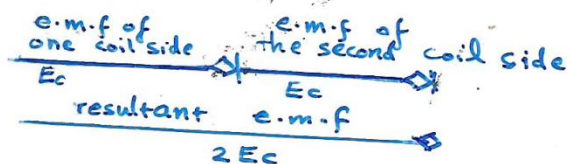


Cooling:- Due to large current carrying capacity of the synch. m/c's. stator windings generate large amount of heat due to I^2R losses. Therefore we have to cool the armature and field windings.

- * For small size machine air is used for cooling.
- * For medium size m/c air (forced air) or water cooled system are used.
- * for large size m/c, water, hydrogen or helium coolants are used.
- * In Iraq most of the thermal power station hydrogen is used as a coolant agent.

As coolant agent

e.m.f's for full-pitched coil:

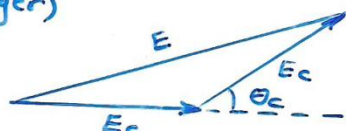


e.m.f.s. for short-pitched coil

$$\theta_c = \frac{180}{g} \times S_c \quad (g \text{ is integer})$$

where S_c = shorted slots.

$$E = 2 E_c \cos \frac{\theta_c}{2}$$



The pitch factor K_p is:

$$K_p = \frac{\text{vector sum of the induced e.m.f.s per coil}}{\text{arithmetic sum of the induced e.m.f.s per coil}} = K_c \text{ (chording factor).}$$

$$\therefore K_p = \frac{E}{2E_c} = \frac{2E_c \cos \frac{\theta_c}{2}}{2E_c} = \cos \frac{\theta_c}{2} = K_c$$

For the previous example, the coil is shorted by $1/6$ pole-pitch i.e. by $\theta_c = \frac{180}{6} = 30^\circ$

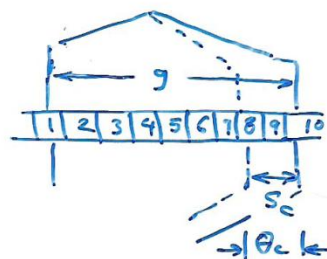
$$\therefore K_p = \cos \frac{30}{2} = \cos 15 = 0.966.$$

Example: Calculate the pitch factor for a synch. m/c stator having 4-pole 36 slots, coil span 1 to 8 slots.

$$\text{Solution: } g = \frac{S}{2P} = \frac{36}{4} = 9.$$

for full-pitched winding
coil span 1 to 10
 $S_c = 2$ slots

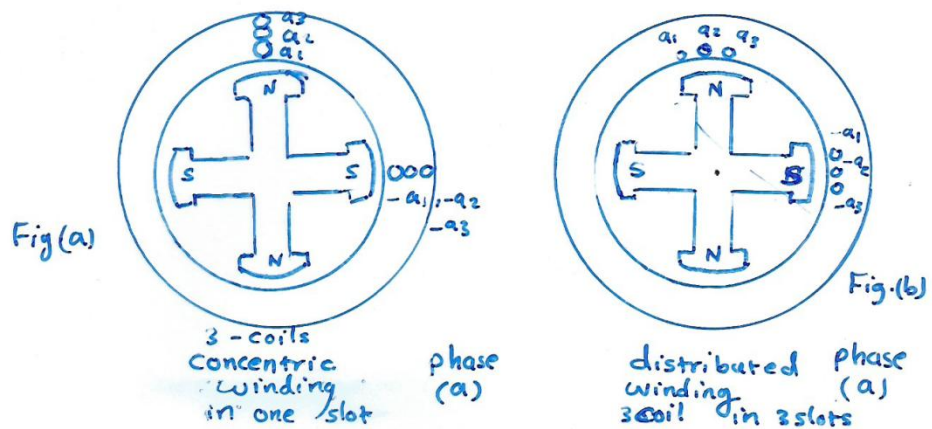
$$\therefore \theta_c = \frac{180}{g} \times S_c = \frac{180}{9} \times 2 = 40^\circ$$



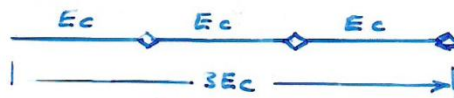
$$\therefore K_p = \cos \frac{\theta_c}{2} = \cos \frac{40}{2} = \cos 20 = \underline{\underline{0.94.}}$$

Distribution and Winding factors (K_d & K_w)

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Consider a 4-pole alternator shown in Figure above if the phase a has three concentric coils layed in one slot, then the total e.m.f induced in the three coil sides of the coils would be the vector sum of the three e.m.f.s. i.e $3E_c$, where E_c is the e.m.f induced in one coil side.



In practice the coil of one phase is distributed as shown in Fig. b above in n slots per pole. i.e we have m slots/phase/pole. Therefore the angular displacement between the slots, α , is

$$\alpha = \frac{180}{\text{No. of slots/pole}} = \frac{180}{n}$$

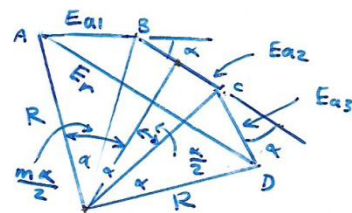
$$BC = E_{a2} = (R \sin \frac{\alpha}{2}) \times 2$$

$$= 2R \sin \frac{\alpha}{2} = E_c$$

$$\text{Arithmetic sum is} = m E_c$$

$$= m \times 2R \sin \frac{\alpha}{2}$$

$$\text{Their vector sum } E_r = 2R \sin \frac{m\alpha}{2}$$



The distribution factor is:

$$K_d = \frac{\text{vector sum of e.m.f.s}}{\text{arithmetic sum}} = \frac{2R \sin m \frac{\alpha}{2}}{m \times 2R \sin \frac{\alpha}{2}}$$

$$= \frac{\sin \frac{m\alpha}{2}}{m \sin \frac{\alpha}{2}}$$

Example: Calculate K_d for a 4-pole 36 slot machine with 3 slot/phase/pole.

Solution:

$$\text{No. of slots/pole } n = \frac{36}{4} = 9.$$

$$m = 3 \text{ slot/phase/pole (given)}$$

$$\alpha = \frac{180}{n} = \frac{180}{9} = 20^\circ.$$

$$\therefore K_d = \frac{\sin \frac{3 \times 20}{2}}{3 \sin \frac{20}{2}} = \underline{\underline{0.96}}.$$

* The distribution factor K_d and pitch factor K_p may sometimes combined as one factor called the winding factor K_w . Where:

$$\boxed{K_w = K_d \cdot K_p}.$$

E.M.F EQUATION FOR SYNCH. MACHINE

The e.m.f equation of a synch. machine, i.e the induced e.m.f per phase is exactly the same equation as the e.m.f equation of a transformer.

$$e_a = \omega N \Phi \sin \omega t \quad \text{for full-pitch m/c.}$$

$$= E_{\max} \sin \omega t.$$

$$\text{where } E_{\max} = \omega N \Phi = 2\pi f N \Phi.$$

$$\& E_{r.m.s} = \frac{2\pi f N \Phi}{\sqrt{2}} = 4.44 f N \Phi \quad \text{Volts.}$$

In general for any type of synch. m/c:

$$E_{r.m.s} = 4.44 K_p \cdot K_d f \Phi N$$

or $E_{r.m.s} = 4.44 K_w f \Phi N$ Volts

Example: A 3-phase, 8-pole star-connected alternator running at 750 r.p.m. having the following data:

Φ = flux per pole = 55 mWb.

S = total number of slots = 72.

number of conductors per slot = 10

distribution factor $K_d = 0.96$

Assuming full-pitch coils, calculate the no load terminal voltage of the alternator.

Solution: Here $K_p = 1$, $K_d = 0.96$.

$$f = \frac{n_p}{120} = \frac{8 \times 750}{120} = 50 \text{ Hz.}$$

$$\Phi = 55 \times 10^{-3} \text{ Wb.}$$

$$\begin{aligned} Z &= \text{number of conductors / phase} \\ &= \frac{\text{Total No. of conductors}}{3} = \frac{72 \times 10}{3} = 240. \end{aligned}$$

$$N = \frac{Z}{2} = \frac{240}{2} = 120.$$

$$\begin{aligned} \therefore E_{a.r.m.s} &= 4.44 K_p \cdot K_d f \Phi N \quad \text{per phase} \\ &= 4.44 \times 1 \times 0.96 \times 50 \times 55 \times 10^{-3} \times 120 = 1407 \text{ V.} \end{aligned}$$

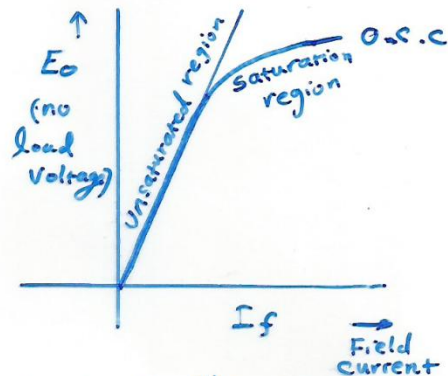
$$\text{Line voltage} = \sqrt{3} E_{a.r.m.s} = 1407 \times \sqrt{3} = 2435 \text{ V.}$$

The O.C.C. (open circuit ch/s) of a synch. Generator 12.

When a synch. m/c is driven as a generator with constant speed, the generated no load e.m.f as a function of the field current I_f is known as the open circuit characteristic of the synch. generator and is shown in Figure below:

E_o = no load voltage
 I_f = field current.

$$E_o = f(I_f).$$



Synchronous Impedance

If we short-circuit the armature at the terminals keeping I_f constant and measure the armature phase current I_a . Thus the entire internal voltage E is dropped across the internal impedance of the machine due to:

(a) drop due to armature resistance $I_a R_a$.

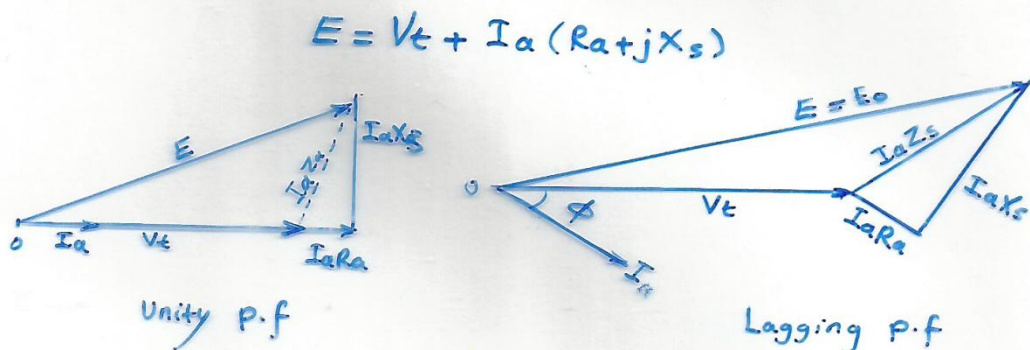
(b) drop due to leakage reactance $I_a X_s$

where X_s is called the synchronous reactance

$$\text{Thus } E = I_a Z_s = I_a (R_a + jX_s)$$

where $Z_s = R_a + jX_s$ is the synch. impedance

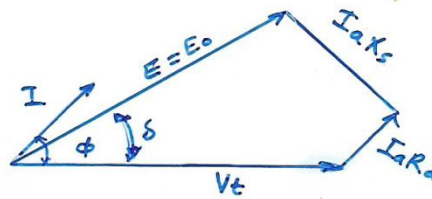
If the generator operates at terminal voltage V_t while supplying a load current I_a (armature current) then:



Vector diagrams of a loaded Alternator

Vector diagram
for leading P.f:

δ is the angle
between E and V_t .
= power angle.



Example: A 10000 KW, 11 KV, 3-phase star-connected alternator supplying full load at 0.85 P.f lagging. The armature resistance R_a per phase is 0.1Ω and the synchronous reactance is 0.66Ω per phase. calculate the generated e.m.f.

Solution:

$$\text{Full-load current } I_{F.L.} = \frac{10000 \times 10^3}{\sqrt{3} \times 11000 \times 0.85} = \frac{\text{KW} \times 10^3}{\sqrt{3} V_L \cos \phi}$$

$$= 618 \text{ A} = I_a$$

$$\text{terminal phase Voltage} = \frac{V_L}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6350 \text{ V.}$$

The drops are:

$$I_a R_a = 618 \times 0.1 = 61.8 \text{ V.}$$

$$I_a X_s = 618 \times 0.66 = 408$$

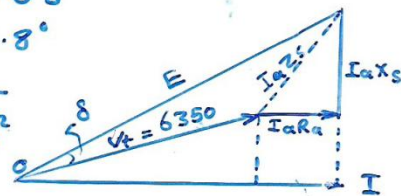
$$\phi = \cos^{-1}(0.85) = 31.8^\circ$$

$$E = \sqrt{(V_t \cos \phi + I_a R_a)^2 + (V_t \sin \phi + I_a X_s)^2}$$

$$= \sqrt{(6350 \times 0.85 + 61.8)^2 + (6350 \times 0.527 + 408)^2}$$

$$= 6625 \text{ V.}$$

$$\therefore \text{Line e.m.f} = \sqrt{3} E = \sqrt{3} \times 6625 = 11480 \text{ V.}$$



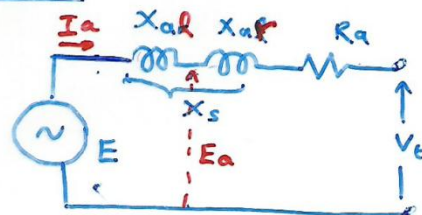
Equivalent circuit per phase of a round-rotor (cylindrical rotor) synch. m/c:

$$X_s = X_{ar} + X_{al}$$

X_{ar} = armature reaction reactance

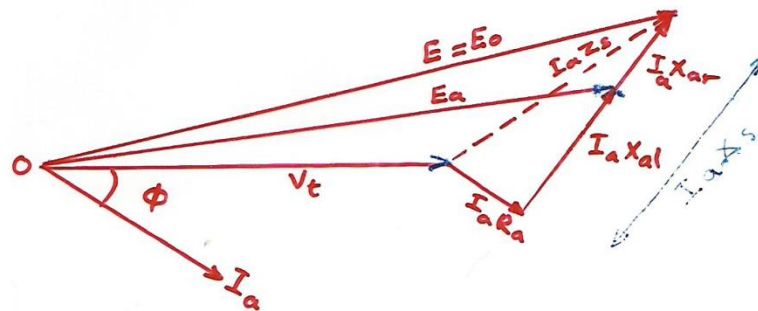
X_{al} = leakage reactance

R_a = effective stator (armature) wdgs resistance $\approx 1.6 R_{d.c.}$



The round-rotor synchronous machine has equivalent circuit as shown in the above figure. The voltage relationships in this circuit are due to:

- (a) the terminal voltage V_t (b) the induced e.m.f E
- (c) the drop due to armature resistance $I_a R_a$
- (d) drop due to leakage reactance $I_a X_{al}$ (e) drop due to armature reaction.



When the alternator is on no-load, the m.m.f in the air-gap is only due to the pole fields. Now, if the alternator is loaded, the armature will carry current, this current will produce its own m.m.f which will interact with the main field. The field in the air gap is therefore a resultant of the two fields. The change in the air gap m.m.f is thus due to armature reaction. The drop in voltage due to armature reaction may be accounted for by assuming the presence of a fictitious reactance X_{ar} in the armature winding. Hence $I_a X_{ar}$ represents the voltage drop due to armature reaction.

The leakage reactance X_{al} and the X_{ar} may be combined to give the synchronous reactance X_s .

$$\text{Hence } X_s = X_{al} + X_{ar}.$$

Thus, the total voltage drop in an alternator under load:

$$= I_a R_a + j I_a X_s = I_a (R_a + j X_s) = I_a Z_s.$$

Z_s = synchronous impedance.

Example A 50 KVA, Y-connected, 440V, 3-phase, 50Hz alternator has armature resistance $R_a = 0.25 \Omega/\text{phase}$. The synchronous reactance is $3.2 \Omega/\text{phase}$ and leakage reactance $X_{al} = 0.5 \Omega/\text{phase}$. The alternator operates at unity power factor. Determine:

- internal e.m.f E_a on load
- internal e.m.f E_0 on no-load.
- Value of synchronous reactance which replace armature reaction (X_{ar}).
- The Voltage regulation on full-load.

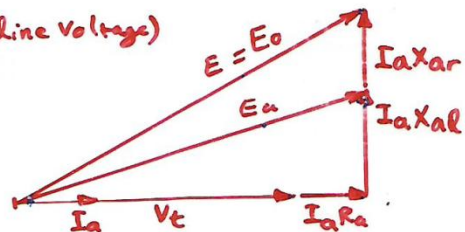
Solution:

The e.m.f E is the vector sum of (i) terminal voltage V_t (ii) $I_a R_a$ and (iii) $I_a Z_s$.

$$V_{t_{ph}} = \frac{440}{\sqrt{3}} = 254 \text{ Volts (phase voltage)}$$

$$I_{a_{F.L.}} = \frac{50,000}{\sqrt{3} \times V_{t_{ph}} \times \cos \phi} \quad (V_{t_{ph}} = \text{line voltage})$$

$$= \frac{50,000}{\sqrt{3} \times 440} = 65.6 \text{ A}$$



$$(a) \quad E_a = \sqrt{(V_{t_{ph}} + I_a R_a)^2 + (I_a X_{al})^2}$$

$$= \sqrt{(254 + 16.4)^2 + (65.6 \times 0.25)^2}$$

$$= 272 \text{ Volts}$$

$$\text{Line voltage} = 272 \times \sqrt{3} = 471 \text{ Volts.}$$

(b) The no load e.m.f $E_0 = E$ is

$$E = E_0 = \sqrt{(V_{t_{ph}} + I_a R_a)^2 + I_a^2 (X_{ar} + X_{al})^2} \quad (c)$$

$$= \sqrt{(254 + (65.6 \times 0.25))^2 + (65.6)^2 (3.2)^2}$$

$$= 342 \text{ Volts}$$

$$(X_{ar} + X_{al} = X_s)$$

$$\therefore X_{ar} = X_s - X_{al}$$

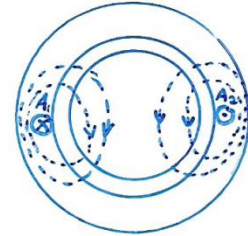
$$\text{Line } E_0 = \sqrt{3} \times 342 = 592 \text{ Volts}$$

$$(d) \% \text{ Reg} = \frac{E_0 - V_{t_{ph}}}{V_{t_{ph}}} \times 100 = \frac{342 - 254}{254} \times 100 = 34.65\%$$

Production of a Rotating Magnetic Field

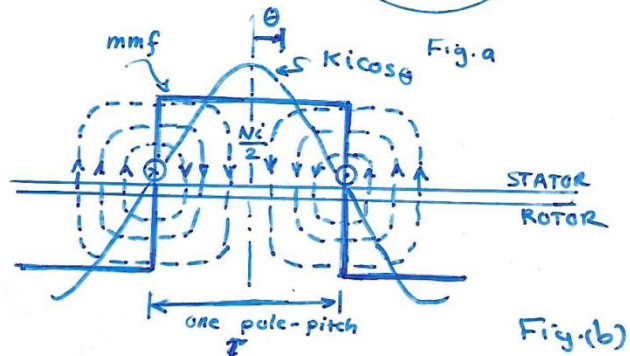
Consider a two-pole single-phase winding as shown in Fig. (a). Assume full-pitch coil AA_2 represent the winding of N turns carrying current (i). Therefore the mmf per pole is $\frac{Ni}{2}$.

This mmf is shown in Fig. (b) has a constant value.



* The flux density B distribution is the same as the mmf distribution.

* The mmf waveform is rectangular. It has to be resolved into fundamental component and a number of higher order harmonics (using Fourier series). However if we neglect the harmonics and only taking the fundamental component [of the mmf ($\frac{Ni}{2}$)]. This component is expressed as



This component is expressed as

$$F = k i \cos \theta$$

where k is a constant and θ is the angle measured w.r.t the magnetic axis of the winding.

Note: If the number of poles = P & $p = \frac{P}{2}$ = number of pole pairs thus for a machine with p pole pairs

$$F = k i \cos p \theta$$

$$\text{If } i = I_m \sin \omega t$$

$$\text{Hence } F = k I_m \sin \omega t \cos p \theta = F_m \sin \omega t \cos p \theta$$

Consider now the stator has a 3-phase winding. As a result the mmf's produced by the three phases are displaced from each other by 120° in time and space.

$$F_a = F_m \sin \omega t \cos p\theta \quad (\text{phase-a})$$

$$F_b = F_m \sin(\omega t - 120^\circ) \cos(p\theta - 120^\circ)$$

$$F_c = F_m \sin(\omega t - 240^\circ) \cos(p\theta - 240^\circ)$$

The resultant m.m.f F_R :

$$F_R = F_a + F_b + F_c = 1.5 F_m \sin(\omega t - p\theta)$$

[Note that: $\sin A \cos B = \frac{1}{2} \sin(A-B) + \frac{1}{2} \sin(A+B)$]

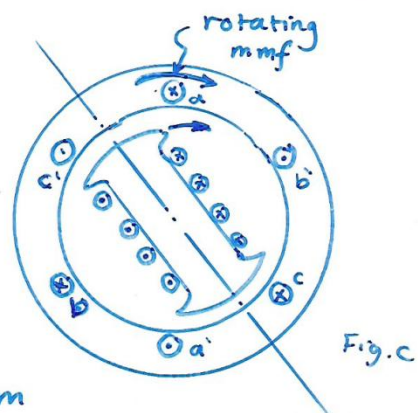
Therefore the resulting magnetic field is a rotating one.

As a conclusion: The stator of a 3-phase synchronous m/c, carrying a three-phase currents, produces a rotating magnetic field in the air gap of the machine.

SYNCHRONOUS MOTOR

Synchronous motor has same construction as synch. generator. The stator is fed from the normal 3-phase supply producing a rotating magnetic flux in the air gap.

The rotor which fed from a d.c. source produces magnetic field of definite polarities (Fig. c), and the rotor will tend to align with the stator field and will tend to rotate with the rotating magnetic field (torque is produced).



Developed power and Output power:

In general, for a practical alternator, $X_s \gg R_a$.
Hence we can neglect the value of R_a .

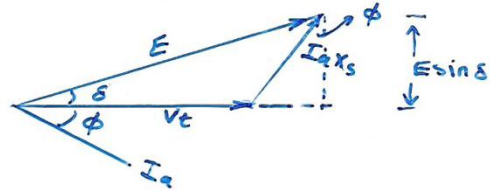
from the vector diagram

Shown:

$$E \sin \delta = I_a X_s \cos \phi \quad \dots (1)$$

$$P_d = V_t I_a \cos \phi \quad \dots (2)$$

$$= \text{developed power} = \text{output power } P_o \text{ since } R_a \approx 0.$$



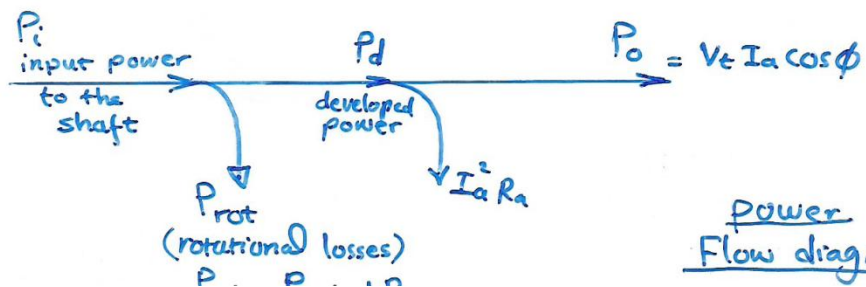
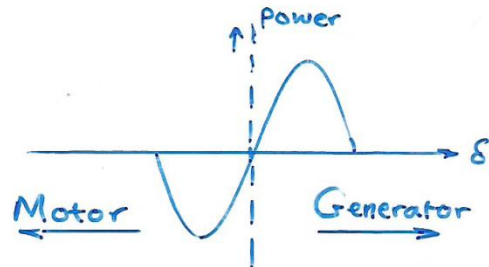
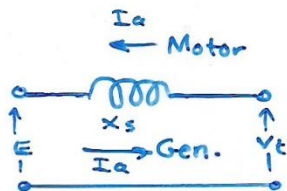
from Eq. (1) $I_a \cos \phi = \frac{E \sin \delta}{X_s}$

$$\therefore P_o = P_d = \frac{V_t E \sin \delta}{X_s}$$

This shows that the output power is proportional to the $\sin \delta$. and the relationship is known as The power-angle Characteristic of a synch. m/c.

Note:

when δ is negative, the m/c will operate as a synch. motor.



Power Flow diagram

$$P_{rot} = P_{e+h} + P_{f+w}$$

eddy current + hyst. friction and windage

Fig. d. represents diagrammatically a 3-phase, star-connected synchronous motor connected to the supply mains and excited by a field winding F in series with a variable resistor R_f .

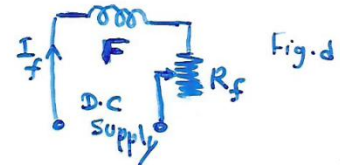
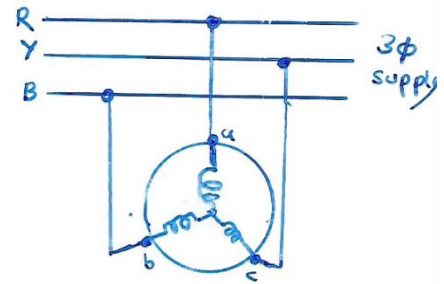


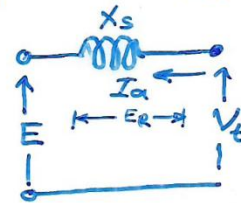
Fig. d

Assume the terminal voltage per phase is V_t . Thus the equivalent circuit of the motor is simply similar to the equivalent circuit of a synch. generator except that power is drawn from the 3-ph main. Therefore

$$V_t = E + I_a X_s$$

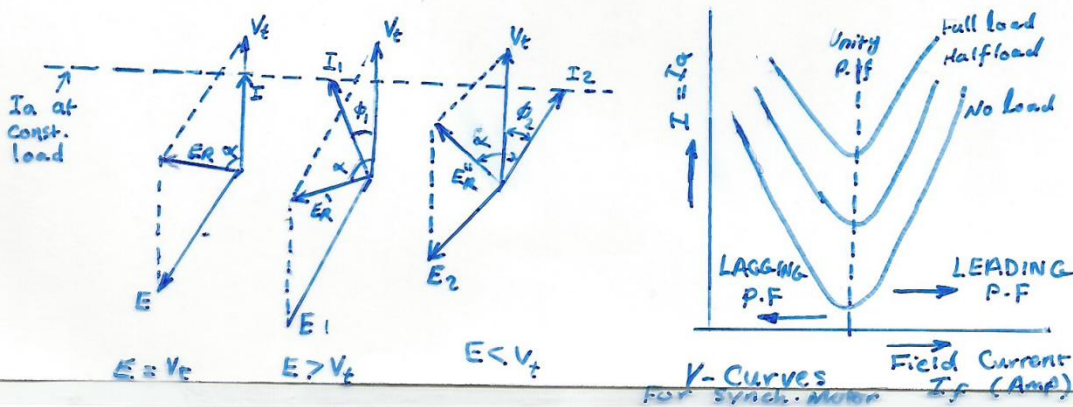
If the motor operates at constant power, thus

$$E \sin \delta = I_a X_s \cos \phi = \text{constant.}$$



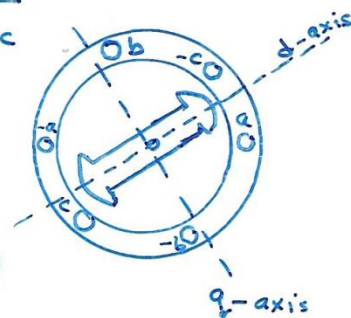
Effect of Varying the Excitation of a synch. Motor

Since E depends on the field current I_f , therefore the value of I_f for which $E = V_t$ (in magnitude) is known as 100% excitation. We shall now discuss the case when I_f is adjusted so that $E < V_t$ (under-excited motor) and when I_f is increased to the point when $E > V_t$ (over-excited motor)



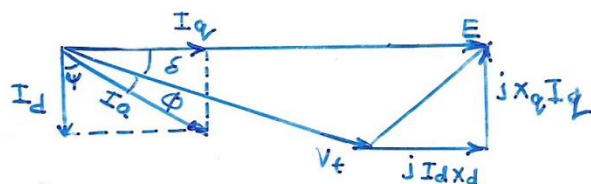
SALIENT-POLE SYNCHRONOUS MACHINES:

The air gap of the salient-pole m/c is not uniform if we compare it with the round-rotor m/c. Therefore it presents two different axes of geometry, one is called the direct-axis (d-axis) corresponding to the axis of the field-pole and the other is called the quadrature axis (q-axis), correspond to the axis of the interpolar space.



Andr  Blondel proposed the two-reaction theory. This theory resolves the armature m.m.f.s into two perpendicular components. One is located along the axis of the rotor salient pole (d-axis) and the other along the quadrature axis (q-axis). Correspondingly we may define X_d (d-axis synch. reactance) and X_q (q-axis synch. reactance). If the armature resistance is neglected, the vector diagram of a salient-pole m/c is:

I_a is resolved into two fictitious components I_d & I_q .



From the vector diagram shown, we have:

$$I_d = I_a \sin(\delta + \phi) \quad I_q = I_a \cos(\delta + \phi)$$

$$V_t \sin \delta = I_q X_q = I_a X_q \cos(\delta + \phi)$$

$$\therefore V_t \sin \delta = I_a X_q [\cos \delta \cos \phi - \sin \delta \sin \phi]$$

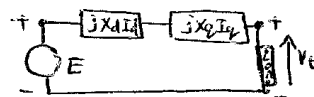
$$\text{or } (V_t \sin \delta + I_a X_q \sin \delta \sin \phi) = I_a X_q \cos \delta \cos \phi$$

$$\sin \delta (V_t + I_a X_q \sin \phi) = I_a X_q \cos \delta \cos \phi$$

$$\therefore \frac{\sin \delta}{\cos \delta} = \tan \delta = \frac{I_a X_q \cos \phi}{V_t + I_a X_q \sin \phi}$$

If δ is known from the above equation (in terms of φ) the generated no load e.m.f E may be found as:

$$E = V_t \cos \delta + I_d X_d \quad \text{or} \quad E = V_t + j X_d I_d + j X_q I_q$$



Calculation of Power

Considering the above phasor diagram and assume R_a is neglected:

$$P_d = V_t I_a \cos \varphi \quad \dots (1)$$

$$I_d X_d = E - V_t \cos \delta \Rightarrow I_d = \frac{E - V_t \cos \delta}{X_d} \quad \dots (2)$$

$$X_q I_q = V_t \sin \delta \Rightarrow I_q = \frac{V_t \sin \delta}{X_q} \quad \dots (3)$$

$$\text{since } \phi + \psi + \delta = 90^\circ$$

$$\therefore \phi = 90 - (\psi + \delta)$$

$$\cos \phi = \cos (90 - (\psi + \delta)) = \sin (\psi + \delta) = \sin \psi \cos \delta + \cos \psi \sin \delta$$

From the phasor diagram: $\dots (4)$

$$\sin \psi = \frac{I_q}{I_a} \quad \& \quad \cos \psi = \frac{I_d}{I_a} \quad \dots (5)$$

subst. (5) in (4) yields:

$$\cos \phi = \sin \delta \frac{I_d}{I_a} + \cos \delta \frac{I_q}{I_a} \quad \dots (6)$$

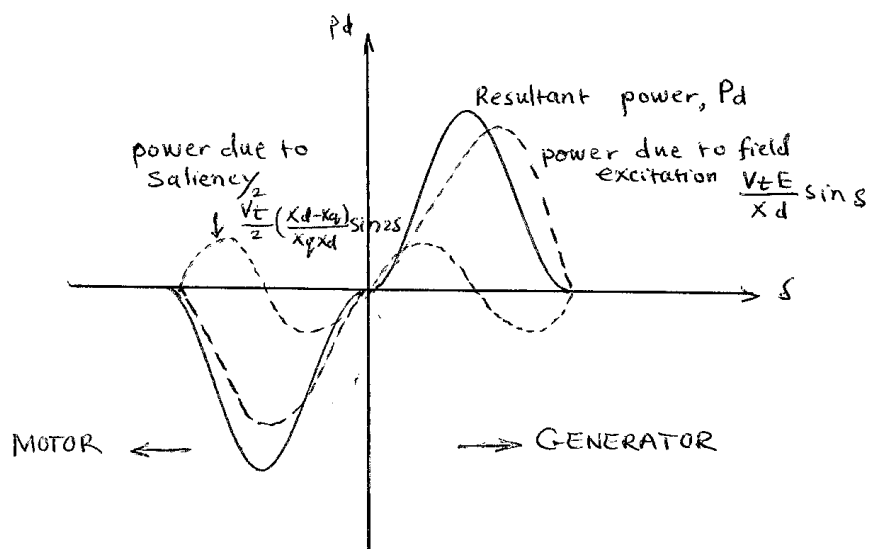
$$\therefore I_a \cos \phi = I_d \sin \delta + I_q \cos \delta \quad \dots (7)$$

Now substituting (2) & (3) into (7) yields:

$$\begin{aligned} I_a \cos \phi &= \left(\frac{E - V_t \cos \delta}{X_d} \right) \sin \delta + \frac{V_t \sin \delta}{X_q} \cos \delta \\ &= \frac{E \sin \delta}{X_d} - \frac{V_t \cos \delta \sin \delta}{X_d} + \frac{V_t \sin \delta \cos \delta}{X_q} \\ &= \frac{E \sin \delta}{X_d} + \frac{V_t}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) 2 \sin \delta \cos \delta \\ &= \frac{E \sin \delta}{X_d} + \frac{V_t}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta \end{aligned}$$

Now the developed power is:

$$P_d = V_t I_a \cos \phi = \frac{V_t E \sin \delta}{X_d} + \frac{V_t^2}{2} \left(\frac{X_d - X_q}{X_d X_q} \right) \sin 2\delta \quad \dots (8)$$



power-angle characteristics of salient-pole m/c.

Note:

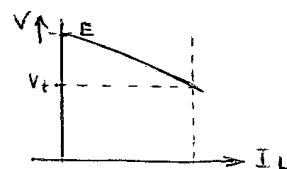
For uniform air gap $X_d = X_q$, thus the second term of equation (8) which represents the power due to saliency becomes zero. Hence eq. (8) will be reduced to $P_d = \frac{V_t E}{X_d} \sin \delta$ which is the equation for round rotor machine given previously.

Voltage Regulation of Synch. Generator (Alternator)

It is defined as the percentage rise in terminal voltage when the load is removed. That is

$$\% \text{ Regulation} = \frac{E - V_t}{V_t} \times 100.$$

With a change in load there is a change in terminal voltage of the alternator, thus the voltage regulation depends on the power factor. It can be positive, zero or negative value.



The PER-UNIT SYSTEM

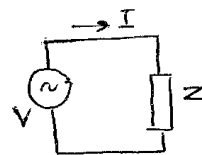
The Per Unit (P.U.) value of any quantity is defined as the ratio of that quantity to an arbitrarily chosen value having the same dimensions and called the base value.

If a single-phase supply of voltage V feeds a current I amperes to a load impedance $Z \Omega$. If V_b (base voltage) is chosen, then the P.U. value of the supply voltage is

$$V_{p.u.} = \frac{V}{V_b}$$

similarly $I_{p.u.} = \frac{I}{I_b}$

(Always we choose
two quantities
 V_b & I_b)



$$\therefore Z_b = \frac{V_b}{I_b} \quad \& \quad Z_{p.u.} = \frac{Z}{Z_b} = \frac{V/I}{V_b/I_b} = \frac{V/V_b}{I/I_b} = \frac{V_{p.u.}}{I_{p.u.}}$$

Thus the p.u. system obeys ohms law (and hence all the other networks laws).

$$\text{Base Volt-Amp } S_b = V_b I_b.$$

$$\& \quad S_{p.u.} = \frac{S}{S_b} = \frac{VI}{V_b I_b} = V_{p.u.} \cdot I_{p.u.}$$

Example:

On a certain synchronous generator, $X_d = 0.9$ P.U. and $X_q = 0.6$ P.U. The machine is operating at full load, 0.8 P.f. lagging. Calculate the value of excitation voltage in terms of the terminal voltage. Calculate also the load angle and the values of direct- and quadrature-axis currents. Neglect resistance and saturation.

Solution

Taking V_t as 1 p.u. voltage, full-load current as 1 p.u. current and referring to the figure:

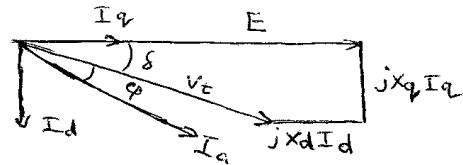
$$V_t \sin \delta = X_q I_q = 0.6 \times I_a \cos (\varphi + \delta)$$

$$\text{In p.u. } \sin \delta = 0.6 (\cos \varphi \cos \delta - \sin \varphi \sin \delta), \quad \text{Since } V_t = 1 \\ = 0.6 (0.8 \cos \delta - 0.6 \sin \delta) \quad \& \quad I_a = 1$$

from which: $1.36 \sin \delta = 0.48 \cos \delta$.

$$\therefore \tan \delta = \frac{0.48}{1.36} = 0.353$$

$$\underline{\underline{\delta = 19.4}}$$



$$E \cos \delta = V_t \cos \delta = X_d I_d = 0.9 \times I_a (\sin(\phi + \delta))$$

In per-unit $E = 1 \cos 19.4 + 0.9 \sin(36.9 + 19.4)$
 $= 0.942 + 0.9 \times 0.833 = 1.69 \text{ p.u.}$

$$I_q = I_a \cos 56.3 = 1 \times 0.555 = 0.555 \text{ p.u.}$$

$$I_d = I_a \sin 56.3 = 1 \times 0.833 = 0.833 \text{ p.u.}$$

If $V_b = 400 \text{ V}$ and $I_b = 100 \text{ A}$.

Thus $E = 1.69 \times 400 = 676 \text{ V.}$ excitation voltage

$$I_q = 0.555 \times 100 = 55.5 \text{ Amp}$$

$$I_d = 0.833 \times 100 = 83.3 \text{ Amp.}$$

Example-1: A 10 kVA, 230, 50Hz 3-phase star-connected R.R. Alternator has an armature resistance $R_a = 0.5 \Omega$ per phase and synch. reactance of 1.2Ω per phase. The alternator is working at full load. Find the power angle and voltage regulation at (a) 0.8 p.f. lagging (b) 0.8 p.f. leading.

Solution:

(a) At 0.8 p.f. lagging:

For round (cylindrical rotor m/c)

$$E = V_t + I_a(R_a + jX_s)$$

$$V_t = \frac{230}{\sqrt{3}} = 132.8 \text{ V/ph. (Reference)}$$

$$\cos \phi = 0.8 \quad \therefore \phi = 36.9^\circ$$

$$I_a = \frac{10 \times 10^3}{\sqrt{3} \times 230} = 25.1 \text{ A}$$

$$\therefore I_a = 25.1 \angle -36.9^\circ \text{ A}$$

$$E = 132.8 \angle 0^\circ + 25.1 \angle -36.9^\circ (0.5 + j1.2) \\ = 161.8 \angle 5.9^\circ \text{ V/ph.}$$

$$\therefore \delta = 5.9^\circ \quad \text{power angle.}$$

$$\% \text{ V.R.} = \frac{161.8 - 132.8}{132.8} \times 100 = 21.8\%$$

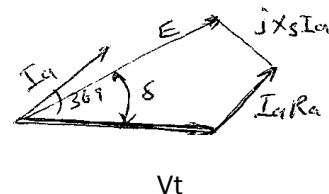
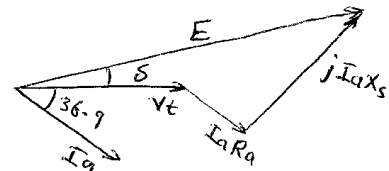
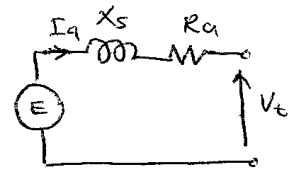
(b) At 0.8 p.f. leading.

$$I_a = 25.1 \angle +36.9^\circ$$

$$\therefore E = 132.8 \angle 0^\circ + 25.1 \angle +36.9^\circ (0.5 + j1.2) \\ = 128.72 \angle 14.225^\circ$$

$$\therefore \delta = 14.225^\circ$$

$$\% \text{ V.R.} = \frac{128.72 - 132.8}{132.8} \times 100 = -3.1\%$$



Example-2. Determine the power factor such that the voltage regulation equal zero on full load for the alternator given in example 1.

Solution:

$$I_a = 25.1 \angle \phi \quad \phi \text{ is unknown.}$$

For zero voltage regulation $E = V_t$.

$$\therefore 132.8 = 132.8 + 25.1 \angle \phi (0.5 + j1.2j).$$

Simplifying the equation:

$$\phi = 29.7^\circ.$$

$$\therefore \text{p.f.} = \cos \phi = \cos 29.7 = 0.864.$$

Since ϕ is positive, therefore the p.f. is leading.

Salient-pole Machine Examples:

Example-3. A 75-MVA, 13.8 kV, three-phase, 8-pole, 60 Hz salient-pole synch. machine has the following d-axis and q-axis reactances: $X_d = 1.0$ p.u. and $X_q = 0.6$ p.u. The synch. generator is delivered rated MVA at rated voltage and 0.866 p.f. lagging. Choose a power base of 75 MVA and a voltage base of 13.8 kV. and let the excitation voltage $E = 1.714$ p.u. & $V_t = 1$ p.u. Find the maximum power and maximum kW that the generator can deliver.

Solution:

The per unit (p.u.) system is defined as:

select Voltage base and current base randomly

$$\therefore \text{P.u. Voltage} = \frac{\text{Actual Voltage}}{\text{Base Voltage}}, \quad \text{P.u. Current} = \frac{\text{MVA}_{\text{base}}}{\text{Voltage}_{\text{base}}}$$

$$\text{P.u. MVA} = \frac{\text{Actual MVA}}{\text{Base MVA}}$$

$$\text{P.u. R or X} = \frac{\text{Actual R or X in ohms}}{\text{Base R or X in ohms}}$$

For this example: $\text{MVA}_{\text{base}} = 75 \text{ MVA}$

$$V_{\text{base}} = 13.8 \text{ KV}$$

Solution:

The p.u. real power that the generator delivers is

$$P = \left[\frac{V_t E}{x_d} \sin \delta + \frac{V_t^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta \right]$$
$$= \frac{(1.0)(1.714)}{1.0} \sin \delta + \frac{(1.0)^2}{2} \left(\frac{1}{0.6} - \frac{1}{1.0} \right) \sin 2\delta$$

For maximum power:

$$\frac{dP}{d\delta} = 1.714 \cos \delta + 0.666 \cos 2\delta = 0.$$

$$1.333 \cos^2 \delta + 1.714 \cos \delta - 0.666 = 0.$$

$$\text{Let } \cos \delta = x$$

$$\therefore 1.333x^2 + 1.714x - 0.666 = 0.$$

Solving by using the law:

$$\therefore \cos \delta = 0.313 \quad \text{or} \quad -1.60 \rightarrow \text{or}$$

$$\therefore \delta = \cos^{-1} 0.313 = 71.8^\circ.$$

$$\therefore P_{\max} = 1.714 \sin 71.8 + 0.333 \sin (2 \times 71.8) = 1.826 \text{ p.u.}$$

$$= 1.826 \times 75 = \underline{\underline{137 \text{ MW}}},$$

Efficiency of a Synch. Machine:

1. GENERATOR: A power flow diagram for a synch. generator is shown in Fig. A. The input power consists of mechanical power supplied to the m/c shaft from the prime mover, and the output is the AC electrical power delivered to the load.

* The DC electrical power supplied to the field winding has not included since this power supplied from a separate DC source.

* The losses of synch. generator are classified in the same manner as in DC machines.

P_{cu} = copper loss = $3 I_a^2 R_a$.

P_{mech} = mechanical losses consist of bearing friction and windage losses.

P_{core} = Core losses consist of hysteresis and eddy current losses.

P_{stray} = Stray losses $\approx 1\%$ of the m/c power output.

The efficiency of the machine is

$$\eta = \frac{P_{output}}{P_{input}} \times 100\% = \frac{P_{output}}{P_{output} + \sum(\text{losses})} \times 100\%.$$

2. MOTOR

Same as the generator with power flow shown in diagram Fig. B

Maximum Power:

Maximum power delivered to the external system occurs when $\delta = 90^\circ$.

$$P_{max} = \frac{3 E V_t}{X_s}$$

Maximum Torque:

$$T_{max} = \frac{P_{max}}{\omega}$$

= pull-out torque.

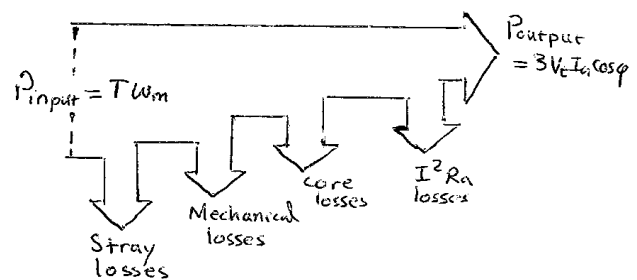
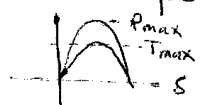


Fig. A

power flow diagram for a generator

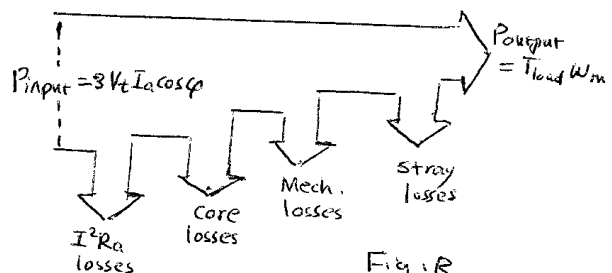


Fig. B

power flow diagram for a motor.

Example: A 200 hP, 2300-V, three-phase, 60 Hz, cylindrical-rotor motor has a synch. reactance of 12Ω per phase and negligible armature resistance. When it delivers its rated output, the motor efficiency is 90% and its power angle δ is 17° . Determine:

- The excitation voltage E .
- The stator current I_a and power factor:

Solution:

- The line-to-neutral terminal voltage is taken as reference phasor, thus.

$$V_t = \frac{2300}{\sqrt{3}} \angle 0^\circ = 1328 \angle 0^\circ.$$

Since the stator copper loss $P_{cu} = 0$.

$$\begin{aligned} \therefore P_{\text{input}} = P_d &= 3 \frac{V_t E}{X_s} \sin \delta = \frac{P_{\text{output}}}{\eta} \\ &= \frac{200 \times 746}{0.90} = 3 \frac{1328 E}{12} \sin 17^\circ. \end{aligned}$$

Thus, the line-to-neutral excitation voltage is

$$E = \frac{200 \times 746 \times 12}{0.9 \times 3 \times 1328 \sin 17^\circ} = \underline{\underline{1708 \text{ V}}}.$$

- the line-to-line excitation voltage is

$$E = \sqrt{3} \times 1708 = 2958 \quad (\text{line-to-line}).$$

- Since the armature resistance is neglected, the stator current is given by:

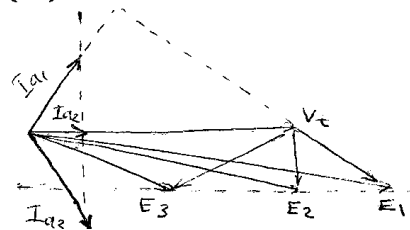
$$I_a = \frac{V_t - E}{jX_s} = \frac{1328 \angle 0^\circ - 1708 \angle -17^\circ}{j12} = 48.8 \angle 31.4^\circ \text{ A}$$

The p.f. is

$$\text{p.f.} = \cos(\angle V_t - \angle I_a) = \cos(0^\circ - 31.4^\circ) = \underline{\underline{0.85}}$$

Note $\cos(x) = \cos(-x)$.

Leading.



Field current effect on synch. motor.

Parallel Operation of Synchronous Generators:

The electricity supply systems of industrialized countries have hundreds of synchronous generators operating in parallel. These generators are interconnected by a network of transmission lines and substations. The main reasons for interconnection are:

1. higher degree of reliability and supply continuity
2. Economy of power system operation
3. Improved operating efficiency of the individual generators.

The point of connection of the generator to the power system is called an infinite bus. The infinite bus can be represented as a voltage source of constant magnitude and constant frequency.

The process of properly connecting a synchronous generator in parallel with the other generators in the power system, or to the infinite bus, is called synchronization. In order that a synchronous generator may be connected in parallel with the system, the following conditions have to be fulfilled.

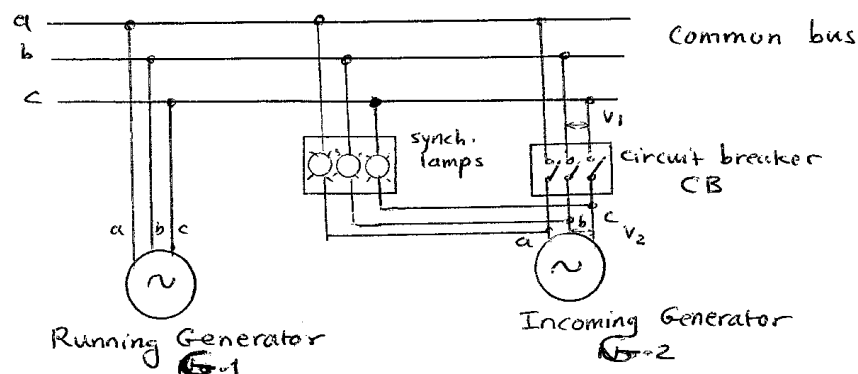
1. The magnitude of the terminal voltage of the incoming generator must be the same as the voltage at the point of interconnection with the power system or infinite bus
2. The frequency of the incoming generator must be the same as the frequency of the power system or infinite bus.
3. The generator must have the same phase sequence as the infinite bus.
4. The phase angles of corresponding phases of the incoming generator and the power system must be equal.

Two generators can be synchronized by two common methods:

1. Using a Synchroscope
2. Using Lamps.

A synchroscope is an instrument having a rotating pointer which indicate whether the m/c (incoming) is slow or fast

Synchronization Using synchronizing lamps



The field rheostat of the incoming generator is adjusted to vary the field current until the generator voltage V_2 becomes equal to, or slightly greater than, the running generator voltage V_1 . If the phase sequences of the two generators are different, the three lamps will brighten up alternately. If the phase sequence is correct, the lamps are all bright or are all dark at the same time.

If the frequencies are slightly different, the three lamps will brighten or darken at the same time. In this case the ~~the~~ speed of the prime mover of the incoming generator is adjusted so that the generator's frequency is the same as that of the running generator, at which time all lamps stay dark. When all four conditions are satisfied simultaneously, the circuit (CB) is closed.

Load Sharing and Circulating Current

1. If G_2 runs at speed less than that of G_1 the phase relationships of their terminal voltages are as shown in Fig. (A).

The resultant voltage V_c will circulate current I_c which has a component of V_1 (in-phase with it). This current will lag V_c by an angle $\psi_c = \tan^{-1} \frac{R_a}{X_s}$.

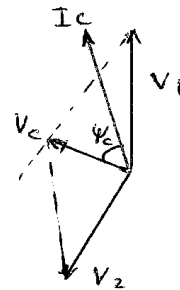


Fig. (A).

$Z_s = R_a + jX_s$ = synch. impedance of each generator.
assuming G_1 & G_2 are identical.

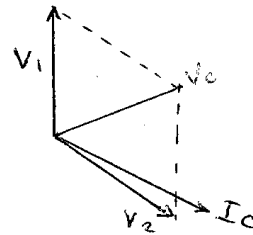
$$\text{and } I_c = \frac{V_c}{2Z_s}$$

- * The in-phase component of I_c with V_1 will act as a load on it and tend to slow it down.
- * The component of I_c in phase opposition to V_2 will aid G_2 . Hence G_2 will operate as a Motor.

2. If G_2 was running faster than G_1 at the instant of synchronization, the phase relationships of the voltages and circulating current become as shown in Fig. (B).

Here, G_2 will function as a generator and will tend to slow down.

G_1 will act as a motor will pick up speed.



Example :- Two- 3-phase, 6.6 kV, Star-connected synch. generators supply a load of 3000 kW at 0.8 p.f. lagging. The synchronous impedance per phase of m/c A is $0.5 + j10 \Omega$ and of machine B is $0.4 + j12 \Omega$. The excitation of machine A is adjusted so that it delivers 150 A at lagging power-factor, and the governors are so set that the load is shared equally between the machines. Determine the current, power factor, induced e.m.f and load angle of each machine.

Solution:

$$\text{For machine A} \quad \cos \phi_A = \frac{1500 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times 150} = 0.874$$

$$\therefore \phi_A = 29^\circ \quad \text{and} \quad \sin \phi_A = 0.485$$

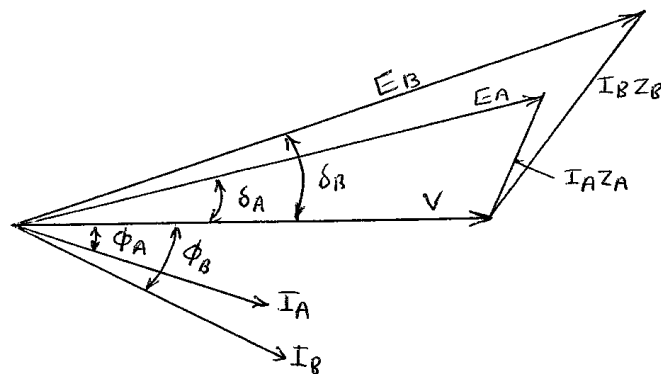
$$I = \text{Total Current} = \frac{P}{\sqrt{3} V \cos \phi} = \frac{3000 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times 0.8} = 328 \text{ A}$$

$$\therefore I = 328(0.8 - j0.6) \text{ A} \\ = 262 - j195 \text{ A}$$

$$I_A = 150(0.874 - j0.485) = 131 - j72.6 \text{ A}$$

$$I_B = I - I_A = 131 - j124.4 \text{ A} \\ = 181 \text{ A}$$

$$\therefore \cos \phi_B = \frac{131}{181} = 0.723 \text{ lagging}$$



Taking V a reference :

$$E_A = V + I_A Z_A = \frac{6.6}{\sqrt{3}} \times 10^3 + (131 - j72.6)(0.5 + j10) \\ = 4.6 + j1.27 \text{ KV.}$$

$$\text{load angle } \delta_A = \tan^{-1} \frac{1.27}{4.6} = 15.4^\circ.$$

$$\text{line value of e.m.f} = \sqrt{3} (\sqrt{(4.6)^2 + (1.27)^2}) = 8.26 \text{ KV.}$$

$$E_B = V + I_B Z_B = \frac{6.6}{\sqrt{3}} \times 10^3 + (131 - j124.4)(0.4 + j12) \\ = 5.35 + j1.52 \text{ KV.}$$

$$\text{load angle } \delta_B = \tan^{-1} \frac{1.52}{5.35} = 15.9^\circ.$$

$$\text{line value of e.m.f} = \sqrt{3} (\sqrt{(5.35)^2 + (1.52)^2}) = \underline{\underline{9.6 \text{ KV}}}$$

Synchronous Machine Parameters (constants) Determination

The synch. m/c parameters can be found from open-circuit and short-circuit tests.

Open-Circuit Test is similar to that of d.c. generator while short circuit test is carried out by short-circuiting the generator terminals and the excitation is varied, a plot of short circuit current vs field current may be obtained. as shown in Fig. below.

Z_s can be obtained graphically as:

Saturated value of Z_s :

$$Z_s = \frac{AC}{AB}$$

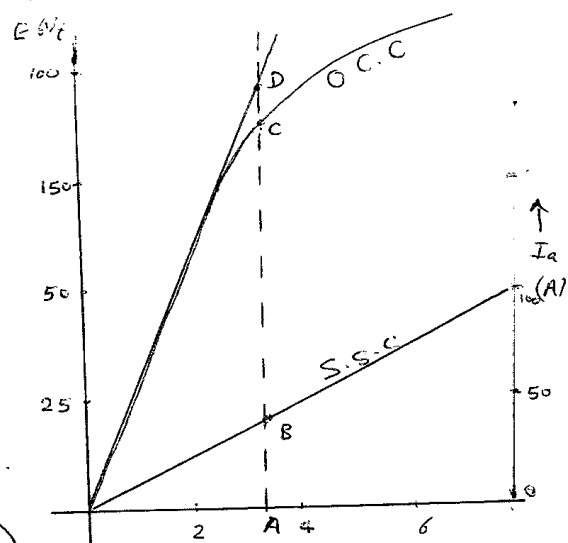
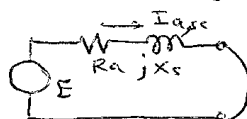
Unsaturated value:

$$Z_s = \frac{AD}{AB}$$

where $Z_s = R_a + jX_s$.

In most synch. m/c's $R_a \ll X_s$.

$$\therefore Z_s = X_s.$$



Example: A 3-ph Y-connected, 2-pole synch. generator is rated at 300 KVA, 480V, 50 Hz, 0.8 p.f lagging. The open-circuit characteristics are given in table-1

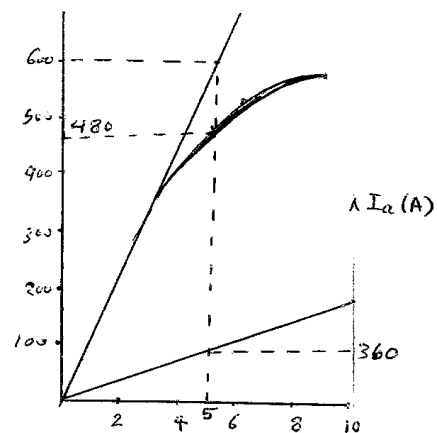
- Determine the unsaturated synch. reactance
- Determine the saturated synch. reactance.

I_f (A)	OCC (VLL)	AG Line (VLL)	SCC (A)
1.0	120		
2.0	240		
3.0	340		
4.0	430		
5.0	480	600	360
6.0	520		
7.0	540		
8.0	550		
9.0	555		
10.0	560		

Solution: For $I_f = 5A$,

a. $X_{s, \text{unsat}} = \frac{(600/\sqrt{3})}{360} = 0.962 \Omega/\text{phase}$

b. $X_{s, \text{sat}} = \frac{(480/\sqrt{3})}{360} = 0.77 \Omega/\text{phase}$



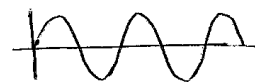
STARTING OF SYNCHRONOUS MOTOR

The torque produced by a synch. motor is

$$T = \frac{P}{\omega} = \frac{E V_t}{X_s} \sin \delta.$$

At standstill $\delta = \omega t$. hence $T = \frac{E V_t}{X_s} \sin \omega t$.

so that the torque produced pulsates at line frequency. The average torque will be zero. therefore the synch. motor is not a self starting motor.



For a synch. motor to start it is necessary that it must be accelerated to synch. speed before the torque reverses. If, however, the motor is brought to speed just below synch. speed, it is possible for the motor to pull into synchronism.

For starting, the motor is equipped a squirrel-cage winding. The field is short circuited to prevent excessive induced voltages and the motor is started as an induction motor, accelerating to just below synch. speed. At this time the field is energized, the motor locks in synchronism and the induction-motor winding has no further effect as long as synch. speed is maintained.